SIMPLIFIED SELF-ABSORPTION CORRECTION FOR ISOTOPES EMITTING WEAK β-PARTICLES

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SUMMARY

A modification of HENDLER's method for adjusting ^{14}C specific-activity values to compensate for self-absorption is proposed. This altered technic permits simplification of computation and reduction of all data to a single reference specific activity. Determination of the approximate specific activity at "zero self-absorption", evaluation of the "slope-constant of self-absorption" for the sample material and ready comparison of specific activities of samples of different materials and thicknesses are additional advantages.

INTRODUCTION

HENDLER[1] refers to loss of observable radioactivity with passage of radiation through the sample as self-absorption, regardless of the mechanism. It is necessary to correct for self-absorption when using isotopic tracers which emit weak beta particles. HENDLER[1-3] has proposed a new method for making these corrections. Other investigators[3,4] have presented arguments against the indiscriminate use of his technic. Errors inherent in sample counting may be greater than the errors in each of various methods of self-absorption correction, so debates concerning relative merit may prove superfluous. The purpose of this paper is to modify HENDLER's treatment, so all values may be readily and easily utilized. It is hoped that this modification will simplify self-absorption corrections for those investigators using his method.

METHODS

HENDLER found that α, the coefficient of self-absorption, is a variable, rather than a constant. It decreases as a function of sample weight, since the spectrum of β-particles is altered as the radiation passes through layers of absorbing substance. HENDLER concluded that, with constant sample area, specific activity appears to follow a hyperbolic curve more closely than the previously accepted negative exponential curve.

All values of radioactivity may be evaluated in terms of an arbitrarily chosen

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reference weight in his method. It is more convenient to calculate the reference specific activity than to attempt to measure the amount of sample material exactly to obtain the desired reference weight. The following symbols are used: m is the weight of the sample in milligrams; S is the specific activity of the sample in counts/min/mg; R is the reference specific activity in counts/min/mg at the reference weight (m_{ref}); I is the observed net counts/min of the sample; I_∞ is the maximum count rate observed at infinite thickness (m_∞); σ is the specific activity in counts/min/mg at infinite thinness (m_0), where the self-absorption is zero, and F is the correction factor for the particular weight (m), where $F = R/S = Rm/I$. HENDLER found by experimentation that plotting F against m yields a straight line from infinite thinness to a weight several times saturation thickness. The slope of the line is R/I and the F-axis intercept is R/σ. The equation which he uses to provide reference specific activity for "various combinations of the data" is

$$R = (1 - G_{1,2})/(1/S_1 - G_{1,2}/S_2)$$

where $G_{1,2}$ is $(m_1 - m_{\text{ref}})/(m_2 - m_{\text{ref}})$ and S_1 and S_2 are the specific activities of m_1 and m_2 materials, respectively.

Slight deviations of experimental values from the expected smooth hyperbolic distribution, however, pose the problem of determining which combinations, if not all, should be used. For example, only six pairs of values for m and S provide fifteen different combinations of two points. Evaluation of data for all fifteen possible combinations of two points in the above expression is tedious. In addition, m_{ref} must be chosen prior to determination of $G_{1,2}$ and R. These disadvantages are easily overcome by altering the technic.

The expression given by HENDLER for stating loss of apparent radioactivity with increasing sample weight is

$$I = (I_\infty Rm)/(Rm + I_\infty R/\sigma)$$

which may be rewritten as $R/S = Rm/I_\infty + R/\sigma$. Division by R simplifies the expression to $1/S = (1/I_\infty)m + 1/\sigma$. This is an expression for a hyperbola which has asymptotes $S = 0$ and $m = -I_\infty/\sigma$ and which crosses the S-axis at σ. Substitution of y for $1/S$, c for $1/I_\infty$ and d for $1/\sigma$ yields the formula of the straight line $y = mc + d$. The slope of the line is c (or $1/I_\infty$), the y-intercept is d (or $1/\sigma$) and the m-intercept is $-I_\infty/\sigma$ (Fig. 1).

LEGENDRE's method of least squares[5] is used to write the two normal equations as follows:

$$c\sum_{j=1}^{f} m_j^2 + d\sum_{j=1}^{f} m_j = \sum_{j=1}^{f} m_j y_j \qquad \text{or}$$

$$(m_1^2 + m_2^2 + \cdots + m_f^2)\,c + (m_1 + m_2 + \cdots + m_f)\,d = m_1y_1 + m_2y_2 + \cdots + m_fy_f$$

$$\text{and} \qquad c\sum_{j=1}^{f} m_j + (f)d = \sum_{j=1}^{f} y_j \qquad \text{or}$$

$$(m_1 + m_2 + \cdots + m_f)\,c + (f)d = y_1 + y_2 + \cdots + y_f$$

where f is the number of samples.

All experimental data for m and y from a given labelled material are substituted into the normal equations. Simultaneous solution of these two expressions yields values of c and d. A linear plot for all data may thus be described by the equation $y = mc + d$. Substitution of any desired value for m yields y or $1/S$ at that particular sample weight.

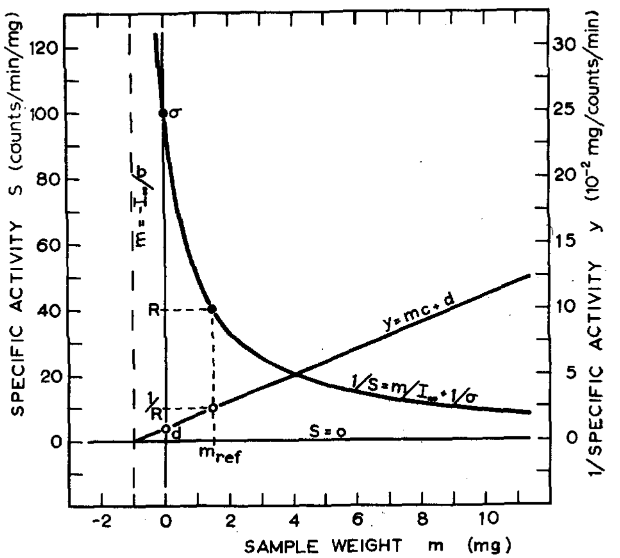

Fig. 1. Specific activity S (counts/min/mg) and 1/specific activity y (10^{-2} mg/counts/min) *versus* sample weight m (mg). Left ordinate is for hyperbola. Right ordinate is for line. ●—●, points (m, S). ○—○, points (m, y). Assigned values represent a theoretical experiment.

To facilitate understanding of the above simplified correction technic, data supplied by HENDLER* have been corrected for self-absorption by this method. Table I shows HENDLER's data. The y column has been added for clarity.

$$\Sigma m = 355.1 \qquad \Sigma m^2 = 44025.87$$

$$\Sigma y = 0.22562122 \qquad \Sigma my = 25.529890353.$$

Thus, the normal equations are:

$$44\,025.87\,c + 355.1\,d = 25.529890353$$

and

$$355.1\,c + 10\,d = 0.22562122$$

Simultaneous solution of the normal equations yields: $c = 0.0005576117352$ and $d = 0.002761329$. When the reference weight (m_{ref}) equals 10 mg, then $y = mc + d$ becomes $y_{\text{ref}} = 10c + d$ or 0.008337446. $R = 1/y_{\text{ref}} = 119.9408068$ or 119.9 counts/min. This single value of R represents the specific activity of HENDLER's $BaCO_3$ at the theoretical weight of 10 mg. It is based upon all ten sets of data. Since the value of 119.9 counts/min for R as calculated above is based upon the mathematical determination of the line best describing the data points, it probably lends additional accuracy to HENDLER's value of 118 counts/min (see ref. 6).

* The author is deeply indebted to R. W. HENDLER who kindly consented to the use of data from Table I of his original article[1]. Values for thickness are taken from a revised reprint in which errors in the original printing had been corrected. Thickness values are not required for the present treatment.

TABLE I

ROBINSON FLOW GAS COUNTER

$BaCO_3$; R at 10 mg; S (HENDLER) 118.

Thickness (mg/cm²)	Weight (mg) m	Observed specific activity (counts/min) S	Reciprocal of specific activity (1/counts/min) y	Specific activity corrected to R = 10 mg with F by HENDLER	Error (%) according to HENDLER
1.23	1.9	234	0.004 273 50	119	+ 1
2.33	3.6	215	0.004 651 16	127	+ 8
2.86	4.4	185	0.005 405 41	118	0
5.97	9.2	128	0.007 812 50	120	+ 2
21.7	33.4	48	0.020 833 33	119	+ 1
59.5	91.7	19	0.052 631 58	118	0
2.47	3.8	194	0.005 154 64	116	— 2
4.55	7.0	149	0.006 711 41	120	+ 2
120	184.6	9.4	0.106 382 98	114	— 3
10.1	15.5	85	0.011 764 71	115	— 4

As c and d are known, calculation of specific activities at other sample weights is easy. All values determined in this manner are thus corrected for self-absorption. This technic allows ready comparisons of specific activities of various sample weights or of different materials. Since c is the "slope-constant of self-absorption" and d is the reciprocal of the approximate specific activity at zero sample weight, or zero self-absorption, it would appear that the collection of self-absorption data would be aided if investigators were to publish values of c and d for their isotope-labelled materials.

DISCUSSION

Although he found a straight-line function "from infinite thinness to a weight of several times saturation thickness", HENDLER[1] admitted that his treatment was empirical and still lacked theoretical justification. He qualified his statement by assuming that the increased back scattering and the increased self-absorption due to relatively uneven spread of sample material at low weights would "probably" counterbalance each other. Also, weighing errors were noted to become relatively more significant with decreasing weights.

Reliability of the various measurements at weights approaching infinite thinness is necessarily in doubt, so self-absorption behavior in this region is speculative. The symbol d is at best an approximation of the actual (unknown) value of the reciprocal of specific activity at infinite thinness. It measures the degree of isotopic tagging.

The slope-constant c is related to self-absorption properties of the sample material and is not dependent upon the relative amount of isotope present. Best results are obtained when samples of both relatively heavy and light weights are employed, since c is influenced to a greater extent by experimental errors if the points are closely bunched within a narrow weight range.

Machine calculation is rapid and accurate, allowing simultaneous determination of Σm and Σm^2 and of Σy and Σmy. Graphing is unnecessary.

This modified technic permits simplification of computation and accurate reduction of all data to a single reference specific activity. Determination of the approximate specific activity at "zero self-absorption", evaluation of the "slope-constant of self-absorption" for the sample material and ready comparison of specific activities of samples of different materials and thicknesses are additional advantages.

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REFERENCES

[1] R. W. HENDLER, *Science*, 130 (1959) 772.
[2] R. W. HENDLER, *Science*, 130 (1959) 1701.
[3] J. KATZ AND R. W. HENDLER, *Science*, 131 (1960) 1886.
[4] P. MASSINI, *Science*, 133 (1961) 887.
[5] E. WHITTAKER AND G. ROBINSON, *The Calculus of Observations*, Fourth Edition, Blackie and Son Limited, Glasgow, 1944, p. 211.
[6] R. W. HENDLER, personal communication.